$\square$ Code No. : 14111

## VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD

B.E. (CBCS) IV-Semester Main \& Backlog Examinations, May-2019

## Engineering Mathematics-IV

Time: 3 hours
Max. Marks: 60
Note: Answer ALL questions in Part-A and any FIVE from Part-B

| Q.No. | Stem of the question |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Part-A $(10 \times 2=20$ Marks $)$ <br> 1. |  |  |  | State sufficient conditions for existence of Laplace transform. Give example of a |

2. Find the inverse Laplace transform $\frac{e^{-3 s}}{s+2}$
3. Compute the Fourier sine and cosine transforms of $x$.
4. Given $F\left(e^{-x^{2}}\right)=\sqrt{\pi} e^{-s^{2} / 4}$, find Fourier transform of $e^{-4(x-3)^{2}}$
5. State Scaling and Shifting properties of $\mathbf{Z}$ transform
6. Find $Z\left\{n a^{n}\right\}$
7. What are the sufficient conditions for a function to be analytic?
8. Show that the function $u(r, \theta)=r^{2} \cos 2 \theta$ is harmonic.
9. Obtain Taylor series of $\frac{1}{z+1}$ about $z=1$.
10. Define (i) zero and (ii) singular point of a function. Give one example each.

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\text { Part-B }(5 \times 8=40 \text { Marks })
$$

11. a) Solve the differential equation $y^{\prime \prime}+y=e^{t} \sin t, y(0)=0, y^{\prime}(0)=0$
b) Derive Laplace transform of derivative of a function.
12. a) Find the Fourier transform of $f(x)=x^{2},|x|<a ; 0,|x|>a$
b) Solve the integral equation $\int_{0}^{\infty} f(x) \cos \alpha x=e^{-\alpha}$
13. a) Determine $f_{0}, f_{1} ; f_{2}$ in the sequence $\left\{f_{n}\right\}$, when $F(z)=\frac{3 z^{2}-4 z+7}{(z-1)^{3}}$
b) Solve $u_{n+2}-2 u_{n+1}+u_{n}=3 n+5$, using Z-transforms.
14. a) Find the analytic function

$$
f(z)=u+i v, \text { if } u-v=(x-y)\left(x^{2}+4 x y+y^{2}\right)
$$

b) Using the Cauchy integral theorem and its extension evaluate $\oint_{C} \frac{d Z}{Z(z+2)}$, where C is the rectangle containing the points $\mathrm{Z}=0$ and $\mathrm{Z}=-2$ inside it.
15. a) Find the series expansion of the function $\frac{1}{\left(z^{2}-1\right)}$ in the region
(i) $|z-1|<2(i i)|z-1|>2$
b) Evaluate $\oint_{C} \frac{e^{z}}{\cos \pi z} d z, C:|z|=1$
16. a) Find the inverse Laplace transform of $\frac{1}{\left(s^{2}+a^{2}\right)^{2}}$
b) Find $f(x)$ if its Fourier sine transform is $\frac{1}{s} e^{-a s}, a>0$
17. Answer any two of the following:
a) Use convolution theorem to evaluate $\boldsymbol{Z}^{\mathbf{1}}\left\{\frac{z^{2}}{(z-a)^{2}}\right\}$
b) Evaluate the integral $\oint_{C} \frac{e^{z} d z}{z^{2}(z+1)^{3}}$, where $\mathrm{C}:|z|=2$.
c) Evaluate the integral $\int_{0}^{2 \pi} \frac{d \theta}{1-2 a \cos \theta+a^{2}}$ where $a$ is a complex constant and $|\mathrm{a}|<1$.
$\left.\left\lvert\, \begin{array}{llll}4 & 1 & 1 & 1 \\ 4 & 4 & 1 & 1 \\ 4 & 2 & 2 & 1 \\ 4 & 4 & 3 & 1 \\ 4 & 3 & 3 & 1\end{array}\right.\right]$

M: Marks; L: Bloom's Taxonomy Level; CO: Course Outcome; PO: Programme Outcome

| S. No. | Criteria for questions | Percentage |
| :---: | :--- | :---: |
| 1 | Fundamental knowledge (Level-1 \& 2) | 58.75 |
| 2 | Knowledge on application and analysis (Level-3 \& 4) | 41.25 |
| 3 | *Critical thinking and ability to design (Level-5 \& 6) <br> (*wherever applicable) |  |

